

Chiral Symmetry and Fermion Doubling in the Zero-mode Landau Levels of Massless Dirac Fermions with Disorder

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The effect of disorder on the Landau levels of massless Dirac fermions is examined for the cases with and without the fermion doubling. To tune the doubling a tight-binding model having a complex transfer integral is adopted to shift the energies of two Dirac cones, which is theoretically proposed earlier and realizable in cold atoms in an optical lattice. In the absence of the fermion doubling, the $n = 0$ Landau level is shown to exhibit an anomalous sharpness even if the disorder is uncorrelated in space (i.e., large K-K' scattering). This anomaly occurs when the disorder respects the chiral symmetry of the Dirac cone.

I. INTRODUCTION

Kicked off by the experimental observation of the graphene quantum Hall effect^{1,2}, fascination with massless Dirac fermions is mounting, where they appear not only in graphene but more generically in various systems such as organic metals³⁻⁵, cold atom systems in optical lattices⁶ and molecular graphene⁷. Among these systems, the number of massless Dirac fermions is always even for solid state materials, which is called the “fermion doubling”^{8,9}. On the other hand, manipulation of Dirac cones into single cones has been theoretically considered¹⁰, which may be realizable in optical lattices where the Hall conductivity is detectable experimentally¹¹.

Inspired by these, we explore here the effect of disorder for the Landau levels of massless Dirac fermions, in particular in the absence of the fermion doubling. Specifically, the effect on the $n = 0$ Landau level, which is essential to the anomalous Hall effect of massless Dirac fermions, is examined from the viewpoint of the fermion doubling and the symmetry of the system. In our previous work¹², we have shown that the $n = 0$ Landau level of the honeycomb lattice (graphene) becomes anomalously sharp even in the presence of disorder, if the disorder respects the chiral symmetry and is spatially correlated over a distance exceeding a few lattice constants. Conversely, we have also pointed out that when the disorder is spatially uncorrelated, the $n = 0$ Landau level is broadened just like the other Landau levels, even if the chiral symmetry is respected by the disorder.

To explore the single-to-double Dirac cone crossover, here we consider a two-dimensional lattice model having two Dirac cones which are shifted in energy with each other as in the model proposed by Watanabe et al.¹⁰. We examine numerically the effect of disorder with this lattice model, and have found that, even if the disorder is uncorrelated in space, the $n = 0$ Landau levels start to become anomalously sharp as the two Dirac cones are energetically shifted. Notably, the anomalous sharpness in the absence of the fermion doubling occurs when the disorder respects the chiral symmetry for each cone. In fact, a potential disorder, which breaks the chiral symmetry, washes out the sharpness.

II. MODEL AND NUMERICAL RESULTS

We adopt here a two-dimensional square lattice with the following nearest-neighbor (NN) t and the next nearest-neighbor (NNN) t' transfer integrals,

$$H = \sum_{\mathbf{r}} -tc_{\mathbf{r}+\mathbf{e}_x}^\dagger c_{\mathbf{r}} + (-1)^{n_x+n_y}tc_{\mathbf{r}+\mathbf{e}_y}^\dagger c_{\mathbf{r}} + it'(c_{\mathbf{r}+\mathbf{e}_x+\mathbf{e}_y}^\dagger c_{\mathbf{r}}) + \text{H.c.}, \quad t, t' \in \mathbf{R} \quad (1)$$

where $\mathbf{r} = (n_x, n_y)$ denotes the lattice points and $\mathbf{e}_x = (1, 0)$ ($\mathbf{e}_y = (0, 1)$) the unit vector in x (y) direction with all lengths measured in units of the lattice constant. To realize shifted Dirac cones some transfer energies have to be complex, and here the NNN transfer is pure imaginary. Although complex transfer integrals may seem unrealistic, they can be realized in cold atoms in optical lattices^{11,13}. In the absence of a magnetic field, the Hamiltonian in the momentum space is expressed as

$$H(\mathbf{k}) = \begin{bmatrix} 2t' \sin k_2 & \Delta(\mathbf{k}) \\ \Delta^*(\mathbf{k}) & 2t' \sin k_2 \end{bmatrix} \quad (2)$$

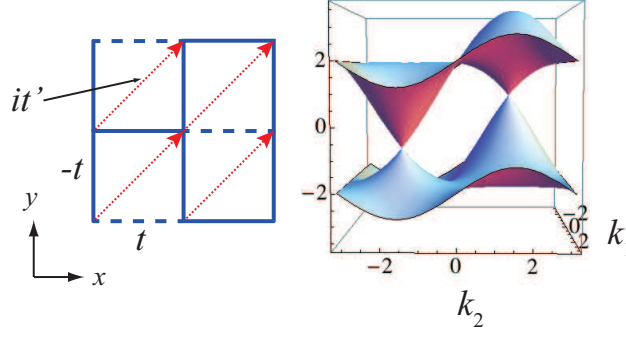


FIG. 1: Left: The present lattice model. Right: Energy dispersion in the k_1 - k_2 plane for $t' = 0.4t$.

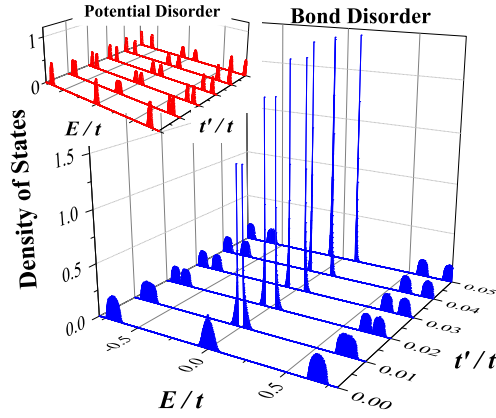


FIG. 2: Density of states as function of the Fermi energy E for various values of t' (\propto the energy shift of two Dirac cones) averaged over 5000 samples with a system size 30 by 30. The magnetic flux piercing each square plaquette is $0.01(h/e)$ and the strength of disorder $\sigma/t = 0.1$. Inset: Density of states for the same parameters when we replace the bond disorder with a potential disorder with a variance $0.1t$.

where $\Delta(\mathbf{k}) = -t(-1 + e^{ik_1} + e^{ik_1+ik_2} + e^{ik_2})$ with $k_1 = \mathbf{k} \cdot \mathbf{e}_1$ and $k_2 = \mathbf{k} \cdot \mathbf{e}_2$ with the primitive vectors taken to be $\mathbf{e}_1 = \mathbf{e}_x - \mathbf{e}_y$ and $\mathbf{e}_2 = \mathbf{e}_x + \mathbf{e}_y$. In this model, we have two Dirac cones at $(k_1, k_2) = \pm(\pi/2, -\pi/2)$ with energies $\pm 2t'$, so that the two Dirac cones are shifted in energy from each other when the strength t' is non-zero. A magnetic field is introduced by taking the Peierls substitution $t(t') \rightarrow t(t')e^{-2\pi i\theta(\mathbf{r})}$, where the summation of the phases $\theta(\mathbf{r})$ along a closed loop is equal to the magnetic flux enclosed by it in units of h/e . We introduce a random component $\delta t(\mathbf{r})$ for the NN transfer integral $t(\mathbf{r}) = t + \delta t(\mathbf{r})$ that has a gaussian distribution with variance σ and is *uncorrelated* in space $\langle \delta t(\mathbf{r}) \delta t(\mathbf{r}') \rangle = \sigma^2 \delta(\mathbf{r} - \mathbf{r}')$. Still, this disorder preserves the chiral symmetry for each Dirac cone. The bond disorder considered in our previous work¹² is equivalent to the present disorder in the limit of zero correlation length.

The numerically evaluated density of states in a magnetic field is shown in Fig.2, where both $n = 0$ and $n = \pm 1$ Landau levels are split as the two Dirac cones are energetically shifted with the increase of t' . Remarkably, the $n = 0$ (zero-mode) Landau level for each cone becomes anomalously sharper with t' even in the presence of disorder, while the $n = \pm 1$ Landau levels remain broadened. For comparison, the density of states calculated for a potential disorder, which breaks the chiral symmetry, shows no such anomaly (Fig.2, inset).

III. SUMMARY AND DISCUSSIONS

We have numerically investigated the effect of disorder on the Landau levels of the massless Dirac fermions with and without the fermion doubling. We have clearly shown that, if the chiral symmetry for Dirac cones is respected,

the zero energy ($n = 0$) Landau level becomes anomalously sharp in the absence of the fermion doubling even when the disorder is uncorrelated in space. Since the shift in energy of the Dirac cones suppresses the mixing between the Dirac points due to the disorder scattering, the present result implies that the broadening reported for uncorrelated disorder in the honeycomb lattice (graphene)¹² is due to the mixing between the two Dirac fermions with opposite chirality.

Acknowledgments

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